Name _____

C. Unit Conversions or Dimensional Analysis

A **unit conversion** is a simple algebraic calculation used to switch a quantity in one particular unit into another unit. For example, mass units can be expressed in grams or milligrams. We can switch between units by choosing the appropriate conversion factor:

$$\frac{1\text{ g}}{1000\text{ mg}} \quad \text{or} \quad \frac{1000\text{ mg}}{1\text{ g}}$$

Which conversion factor to use depends on which unit we want in our final answer, grams or milligrams. For example, to convert 325 mg into grams, we need to set up an equation so that the given unit of (mg) is **algebraically canceled** and the desired unit of (g) remains:

$$325\ \cancel{\text{mg}} \times \left(\frac{1\text{ g}}{1000\ \cancel{\text{mg}}}\right) = 0.325\text{ g}$$

Conversely, to switch grams into milligrams, we use the reciprocal conversion factor. For example, 2.59 g is equal to 2590 mg:

$$2.59\ \cancel{\text{g}} \times \left(\frac{1000\text{ mg}}{1\ \cancel{\text{g}}}\right) = 2590\text{ mg}$$

Dimensional analysis is a technical term for unit conversion, often implying that a particular unit may undergo sequential conversions. For example, to express 525 mg in units of kilograms, one approach uses a two-step process to convert milligrams to grams followed by conversion of grams into kilograms.

Step 1 (mg into g)

$$525\ \cancel{\text{mg}} \times \left(\frac{1\text{ g}}{1000\ \cancel{\text{mg}}}\right) = 0.525\text{ g}$$

Step 2 (g into kg)

$$0.525\ \cancel{\text{g}} \times \left(\frac{1\text{ kg}}{1000\ \cancel{\text{g}}}\right) = 0.000525\text{ kg}$$

A more efficient approach to dimensional analysis makes use of the rules of algebra to combine both steps into a single equation:

$$\underbrace{525\ \cancel{\text{mg}}}_{\text{starting unit}} \times \underbrace{\left(\frac{1\ \cancel{\text{g}}}{1000\ \cancel{\text{mg}}}\right)\left(\frac{1\text{ kg}}{1000\ \cancel{\text{g}}}\right)}_{\text{conversion factor(s)}} = \underbrace{0.000525\text{ kg}}_{\text{final unit}}$$

Name _____

Notice the **three elements of dimensional analysis**: a starting quantity, a desired quantity and conversion factors. The starting quantity and desired quantity are stated (or implied) in the word problem. The conversion factors must be derived base on knowledge of the units of measurement. The **key** to dimensional analysis is proficiency in recognizing these three elements.

When two or more conversion factors are needed, the rules of algebra are followed: $x \bullet \frac{a}{b} \bullet \frac{c}{d} = \frac{(x \bullet a \bullet c)}{(b \bullet d)} =$

Critical Thinking Questions

CTQ 10

What are the three elements needed to set up a calculation for dimensional analysis?

CTQ 11

For each calculation below, draw a **square** around the starting units, draw a **circle** around the conversion factor(s) and draw a **triangle** around the final unit.

a. $22.4\ \text{L} \quad \frac{1000\ \text{mL}}{1\ \text{L}} = 22400\ \text{mL}$

b. $0.78\ \text{cm} \quad \frac{1\ \text{m}}{100\ \text{cm}} = 0.0078 = 7.8 \times 10^{-3}\ \text{m}$

c. $2.5\ \text{ft} \quad \frac{12\ \text{in}}{1\ \text{ft}} \quad \frac{2.54\ \text{cm}}{1\ \text{in}} \quad \frac{10\ \text{mm}}{1\ \text{cm}} = 762 = 7.62 \times 10^{2}\ \text{mm}$

Name _____

CTQ 12

Explain mathematically the result of using the incorrect conversion factor; i.e. what happens to the units if the incorrect reciprocal is used as shown in this example?

$$525 \text{ mg} \quad \frac{1000 \text{ mg}}{1 \text{ g}} \quad = \; ???$$

CTQ 13

What are possible complications that may result if all units are omitted in your calculation?

CTQ 14

In all unit conversions, every number **must** include _____.

CTQ 15

To convert miles per hour into km per second, the calculation requires:

$$25 \; \frac{\text{mi}}{\text{hr}} \longrightarrow \frac{\text{km}}{\text{s}}$$

a. One factor to convert miles into meters and another factor to convert hours into minutes
b. One factor to convert km into miles and another factor to convert seconds into hours
c. One factor to convert miles into km and another factor to convert hours into seconds
d. One factor to convert miles into km another factor to convert hours into minutes and another factor to convert minutes into seconds.
e. Both a and b
f. Both c and d
g. None of the above

Name _____

CTQ 16

Read the word problems carefully. **Do not bother to calculate answers.** Instead, for each problem, **identify the three elements**: starting quantity, the desired (final) unit and the conversion factor(s). Be sure to include proper units with the starting quantity.

Word problem	starting quantity	final unit	conversion factor(s)
Convert 100 centimeters into millimeters.			
How many dL in 15 mL?			
Calculate the mg of gold in a wedding ring that has a mass of 17.5 g			
A 5.0 L vessel containing salt water is cooled to 10°C and the solution is transferred into a holding tank. How many mL of salt water are in the tank?			
Gas mileage for a late model truck is reported as 14 miles per gallon. What is this mileage in km per liter?			
Superman is faster than a speeding bullet which travels at 2700 feet per second. How fast is this in miles per hour?			
A doctor prescribes 1 L of saline solution to be administered intravenously over a two-hour period. How many mL per second is this?			

Name _____

D. More About Dimensional Analysis and Conversion Factors

There are a number of ways to solve any word problem. In the previous example:

$$525\ \cancel{mg} \times \left(\frac{1\ \cancel{g}}{1000\ \cancel{mg}}\right)\left(\frac{1\ kg}{1000\ \cancel{g}}\right) = 0.000525\ kg$$

This calculation required two conversion factors, one quotient for (mg and g) and another for (g and kg) based on the equalities (1000 mg = 1g) and (1000 g = 1 kg), respectively.

However, if one recognizes this equality (1 kg = 1,000,000 mg), then the calculation can be done in a single step with one conversion factor:

$$525\ \cancel{mg} \frac{1\ kg}{10^6\ \cancel{g}} = 0.00525\ kg = 5.25 \times 10^{-3}\ kg$$

Notice the use of scientific notation which eliminates all place-holder zeros.

Both approaches to the word problem are correct since both calculations render the same answer. With complex, multi-step problems, it is recommended to do all calculations in a step-wise fashion unless you know your conversion facts are correct.

CTQ 17

Write a grammatically correct definition of a place-holder zero.

CTQ 18

Convert 2459 km into micrometers. For each calculation, complete each conversion factor and solve the equation.

a. $2459\ km \times \frac{\_\_\_\_\_\ m}{km} \times \frac{\_\_\_\_\_\ cm}{m} \times \frac{\_\_\_\_\_\ mm}{cm} \times \frac{\_\_\_\_\_\ \mu m}{mm} =$

b. $2459\ km \times \frac{\_\_\_\_\_\ \mu m}{km} =$

CTQ 19

Write each answer above in scientific notation.

a. _____ b. _____

Name _____

Exercises

Show calculations and fill in the blank after converting answer to scientific notation.

a. 97.5 m = _____ μm

b. 345 m = _____ cm

c. 2.3×10^{-1} L = _____ μL

d. 1.05 km = _____ mm

e. 24.2 cm^3 = _____ μL

f. 8.89×10^{-6} mg = _____ μg

g. 7.34 mg = _____ kg

h. 75 mL = _____ L

i. 6.53×10^{4} mL = _____ μL

j. 3.5×10^{-4} mg = _____ g

k. 0.25 dL = _____ cc (mL)